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FGM-series
Magnetic Field Sensors

Application Note
Autocalibration algorithms for FGM type sensors

The FGM type sensor has an output in the form of a large rectangular pulse whose period is proportional to the external magnetic field along its principal axis, within its linear range of operation. Unfortunately, since the output cannot have a negative period, this cannot be a direct proportionality and of necessity there has to be a zero field period in the form of a zero-offset large enough to accommodate negative values of magnetic field.

There are then two unknown parameters associated with each sensor, the first being the slope of the relationship between period and magnetic field and the second, the zero-offset or period corresponding to zero magnetic field. Both these parameters need to be taken into account, when using sensor combinations to determine orientation information using the earth's field.

Though an attempt is made to reduce the variation in these parameters no two sensors are alike and some calibration is called for. For small quantities, this calibration is fairly easy to carry out, but in the large scale production of application devices it would be much more desirable to remove the calibration requirement.

There are some circumstances in which continuous autocalibration is possible. One is the two dimensional bolt-down type compass magnetometer, in which the sensors are constrained to rotate in a horizontal plane or at least a fixed plane, which need not necessarily be horizontal. The other is the full three dimensional sensor combination, used to determine the alignment of the earth's field with respect to an orthogonal set of sensor axes.

The only other requirement for this type of autocalibration is that the sensor combination should be in continuous or intermittent motion of some sort. If this is the case it is normally possible to continuously determine and update the values of the two (or three) sensitivities and the two (or three) zero-offsets, using only the readings taken in the normal operation of the orientation determining device.

The fundamental principle behind the method is the fact that the earth's field can be regarded as substantially fixed in both magnitude and orientation and the sum of the squares of the orthogonal field components will remain constant regardless of the orientation of the reference axes. In the three dimensional case, for example, if the field components are h_x, h_y and h_z then :

$$h_x^2 + h_y^2 + h_z^2 = |\mathbf{h}|^2$$

If there are any zero-offsets or unmatched sensitivity variations between sensors then this relationship will not hold true and can be made to indicate the required corrections.

For simplicity, a two dimensional algorithm will be developed first from which the expansion to three dimensions is obvious.

Two dimensional autocalibration method

For an earth field vector, **h,** having orthogonal components h_x and h_y, in the plane of the sensor axes, assume the sensors give output periods of t_x and t_y .

If the sensors have differing sensitivities (slopes of period against field), s_x and s_y and differing zero-offset periods, t_{x0} and t_{y0} , such that:

$$t_x = s_x.h_x / h + t_{x0} \qquad (1)$$

$$t_y = s_y.h_y / h + t_{y0} \qquad (2)$$

where the x and y components of the field are assumed to be divided through by h, the modulus of the field. This effectively converts the field components into their direction cosines, which are independent of the field magnitude.

Applying the condition that $h_x^2 + h_y^2 = h^2$ or more appropriately:

$$(h_x / h)^2 + (h_y / h)^2 = 1$$

we obtain:

$$(t_x - t_{x0})^2 / s_x^2 + (t_y - t_{y0})^2 / s_y^2 = 1 \qquad (3)$$

This is the equation, in t_x and t_y , of an ellipse with centre located at (t_{x0} , t_{y0}), having principal axes s_x and s_y.

All measured pairs of sensor readings, t_x and t_y, must lie on this ellipse and hence any four different points are sufficient to define the ellipse completely.

It must therefore be possible to deduce the centre, (t_{x0},t_{y0}) and the principal axes, s_x and s_y from any four different pairs of sensor readings. Although in theory any four points will do, the precision of calculation with measurements of finite accuracy will be adversely affected if the points are very close together. This should not be a problem with orientation systems which are in constant motion and the algorithm should be designed to wait until it has collected sufficiently different inputs before proceeding to calculate.

If the four points are denoted by (t_{xi},t_{yi}), i = 1,2,3,4 then four equations of type (3) above are available, having the typical form:

$$(t_{xi} - t_{x0})^2 / s_x^2 + (t_{yi} - t_{y0})^2 / s_y^2 = 1$$

Subtracting these from one another successively will yield three equations of the typical form:

$$\frac{(t_{xi}^2 - t_{xi+1}^2) - 2t_{x0}(t_{xi} - t_{xi+1})}{s_x^2} + \frac{(t_{yi}^2 - t_{yi+1}^2) - 2t_{y0}(t_{yi} - t_{yi+1})}{s_y^2} = 0$$

Multiplying each through by s_x^2 and setting $k = s_x^2 / s_y^2$ will yield three equations of the form:

$$a_i t_{x0} + b_i(kt_{y0}) + c_i k = d_i \qquad i = 1,2,3$$

where:

$$a_i = 2(t_{xi} - t_{xi+1})$$

$$b_i = 2(t_{yi} - t_{yi+1})$$

$$c_i = - (t_{yi}^2 - t_{yi+1}^2)$$

$$d_i = (t_{xi}^2 - t_{xi+1}^2)$$

These three equations are linear in t_{x0}, (kt_{y0}) and k and are therefore soluble for these values, using the determinant method or the Gaussian elimination method of solving linear simultaneous equations.

This will immediately provide the values of t_{x0} and t_{y0} .

Finally, the last of the type (3) equations, viz.

$$\frac{(t_{x4} - t_{x0})^2}{s_x^2} + \frac{(t_{y4} - t_{y0})^2}{s_y^2} = 1$$

can be solved for s_x and hence for s_y by setting $s_x^2 = ks_y^2$ and inserting the other known values.

This gives the required sensitivities and zero offsets of the individual sensors.

They can now be used to correct the incoming readings to give valid direction cosines for the orientation calculations, as follows:

For any pair of readings, (t_x,t_y) :

$$h_x / h = (t_x - t_{x0}) / s_x$$

$$h_y / h = (t_y - t_{y0}) / s_y$$

giving the desired corrected values in terms of known constants and measured values.

Then measuring the orientation angle, θ, of the vector **h** in the clockwise direction from the y axis, for example, gives:

$$\theta = \tan^{-1}(h_x / h_y)$$

The calibration can be carried out at whatever intervals are considered appropriate to maintain a suitable compromise of stability and speed of data acquisition.

The FGM sensors are generally stable enough for orientation purposes without continuous recalibration if supplied from a stable voltage source and may only need the initial startup calibration. The technique can be used, however to overcome the effects of drift from any potential source.

The expansion to three dimensions follows the same pattern and ends up defining a three dimensional ellipsoid using six pairs of differing measurements, yielding five simultaneous equations to solve and giving finally three sensitivities and three zero-offsets as corrections.

With this many equations the determinant method of solution is not very efficient and Gaussian elimination is probably the preferred approach.

Practical demonstration

A simple practical demonstration of the mechanics of the algorithm is sometimes useful. This can be carried out by means of a hypothetical experiment in which we assume values for the sensitivities and zero-offsets and back calculate, for selected angles, what the readings from the sensors should look like. The algorithm is then run on the hypothetical readings to show that it can make reasonable estimates of the required corrections.

In practice the techniques for obtaining readings will vary with the individual designers, but a typical method might be to count how many internal processor clock pulses occur during say 128 or 256 incoming sensor pulse periods, using either internal hardware count registers or some software equivalent. This will generate an arbitrary number which is proportional to the sensor period and therefore to the external field strength. The designer will usually arrange that this number ranges over sizes that stay within some register's capacity for convenience, but which is still large enough to provide the desired precision.

For orientation determinations, the field strength itself does not have to be known, so these arbitrary numbers can go directly into the algorithm as they stand.

Using again the two dimensional example, suppose that the sensors have different zero-offsets of :

$$t_{x0} = 2000$$

$$t_{y0} = 1850$$

These are the arbitrary counts obtained at zero field respectively for the two sensors.

Suppose also that the sensitivities differ and can be simulated by slope factors of

$$s_x = 880$$

$$s_y = 740$$

Then we can calculate the expected counts for any angle to, say, the y-axis from:

$$t_x = 880\, h_x / h + 2000$$

$$t_y = 740\, h_y / h + 1850$$

the values of h_x / h and h_y / h being simply the sine and cosine of the chosen angle. These counts now contain the errors caused by the sensitivity and zero-offset factors chosen.

Using four arbitrarily selected angles, θ we can make up a table of the field's direction cosines :

θ	0°	30°	60°	90°
h_x / h	0	0.5	0.86603	1
h_y / h	1	0.86603	0.5	0

From this we can calculate a table of errored readings :

t_x	2000	2440	2762	2880
t_y	2590	2491	2220	1850

Using these readings we can calculate the coefficients of the three simultaneous equations :

i	a_i	b_i	c_i	d_i
1	-880	198	-503019	-1953600

2	-644	542	-1276681	-1675044
3	-236	740	-1505900	-665756

In matrix form then the equations are:

$$\begin{bmatrix} -880 & 198 & -503019 \\ -644 & 542 & -1276681 \\ -236 & 740 & -1505900 \end{bmatrix} \begin{bmatrix} t_{x0} \\ kt_{y0} \\ k \end{bmatrix} = \begin{bmatrix} -1953600 \\ -1675044 \\ -665756 \end{bmatrix}$$

Solving by the determinant method gives :

t_{x0} = 2000.9 (0.05% error)

kt_{y0} = 2604.9

k = 1.4086 giving t_{y0} = 1849.3 (0.04% error)

From the fourth Pythagoras type equation using the last set of readings :

s_y^2 = 548642 and s_y = 740.7 (0.1% error)

from which using $s_x^2 = k\ s_y^2$:

s_x^2 = 772817 and s_x = 879.1 (0.1% error)

The four supposedly unknown parameters have been recovered with a reasonably good accuracy.

Using the corrections above to recalculate the original hypothetical angles from $\theta = \tan^{-1} h_x / h_y$ gives :

h_x / h	-0.001	0.4995	0.8658	1.000
h_x / h_y	1.000	0.8663	0.5005	0.0009
θ	0.06°	29.97°	59.97°	89.95°
Error	0.06°	0.03°	0.03°	0.05°

In any practical situation the errors arising from other causes are likely to be much larger than this, but the demonstration does indicate the efficiency of the technique if other errors are small.

This type of hypothetical experiment can be set up as a computer program and usefully exploited to determine the effects of limited digital measurement precision, non-linearity and non-orthogonality on the success of the algorithms when working in less than perfect conditions.

It should be noted that the errors observed above are entirely the result of the limited precision used in the calculation of the hypothetical inputs, in particular in the trigonometric functions. The algorithm itself is precise when fed with precise input values.